Q1.

10 $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$.		
(i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC} = 4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$	B1	co
Unit vector = $\frac{1}{6}(4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$	M1 A1√	Divides by the modulus. √ on $\overrightarrow{OB}$.
	[3]	
(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$	B1	co
$\overrightarrow{AC} . \overrightarrow{OB} = 8 - 8 - 8 = -8$	M1	Use of $x_1 x_2 + y_1 y_2 + z_1 z_2$
$\lvert\overrightarrow{OB}\rvert = 6$; $\lvert\overrightarrow{AC}\rvert = \sqrt{24}$	M1	Correct method for a modulus.
$-8 = 6 \times \sqrt{24} \times \cos\theta$	M1	Connected correctly provided $\overrightarrow{OB}$, $\overrightarrow{AC}$ used
$\theta = 105.8° \rightarrow 74.2°$	A1	co (accept acute or obtuse)
	[5]	
(iii) $OA = \sqrt{19}$ or $OC = \sqrt{11}$	B1	Used as a length.
Perimeter = 2(√ + √)	M1	
$\rightarrow 15.4$	A1	co (accept 15.3)
	[3]	

Q2.

6 $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$, $\overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$		
(i) (±) $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$	B1	co
(±) $4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$	B1	co
$\overrightarrow{AB}.\overrightarrow{CB} = 16$	M1	Needs to be scalar.
$\overrightarrow{AB}.\overrightarrow{CB} = \sqrt{36}\sqrt{36}\cos\theta$	M1	For product of 2 moduli and cosine
$\theta = 63.6°$	M1 A1	All correct.
	[6]	
(ii) Perimeter $= 6 + 6 + \sqrt{40}$ or $6 + 6 + 6\sin 31.8° \times 2$	M1	Correct overall method for perimeter.
$\rightarrow 18.32$	A1	co
	[2]	

Q3.

4 **(i)** $\overrightarrow{CP} = -6\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$	B1	
$\overrightarrow{CQ} = -6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$	B1	
	[2]	
(ii) Scalar product = 36 + 36 − 6	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
$66 = \lvert\overrightarrow{CP}\rvert\lvert\overrightarrow{CQ}\rvert\cos\theta$	M1	Linking everything correctly
$\lvert\overrightarrow{CP}\rvert = \sqrt{76}$, $\lvert\overrightarrow{CQ}\rvert = \sqrt{81}$	M1	Correct magnitude for either
Angle $PCQ = 32.7°$ (or 0.571 rad)	A1	cao 147.3° converted to 32.7° gets A0
	[4]	

Q4.

5 **(i)** $\overrightarrow{PQ} = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{RQ} = -3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$	B2,1 B1 [3]	Allow B2,1 for either one, B1 for the other.
(ii) $\overrightarrow{PQ} . \overrightarrow{RQ} = -9 + 48 - 9 = 30$ $= \sqrt{54}\ \sqrt{82} \cos ROP$ $\rightarrow ROP = 63.2°$	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct use of modulus All linked correctly co

Q5.

6 **(i)** $2p^2 - 2p + 2 + 12p + 6 \rightarrow 2p^2 + 10p + 8$ $\mathbf{u.v} = 0$ $(p+1)(p+4) - 0 \rightarrow p - -1$ **or** $p - -4$	**M1** **B1** **A1** **[3]**	Correct method for scalar product Scalar product = 0 cao Both solutions required
(ii) $\mathbf{u.v} = 2 + 0 + 18 = 20$ $\lvert\mathbf{u}\rvert = \sqrt{41}$ or $\lvert\mathbf{v}\rvert = \sqrt{13}$ $20 - \sqrt{41} \times \sqrt{13} \times \cos\theta$ oe $\theta - 30.0°$ or 0.523 rads	**M1** **M1** **M1** **A1** **[4]**	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct method for moduli All connected correctly cao

Q6.

2 $\overrightarrow{OA} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$, $\overrightarrow{OB} - \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$, $\overrightarrow{OC} - \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}$.		
(i) $\overrightarrow{AB} - \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ Modulus = $\sqrt{(4+9+36)}$	B1 M1	co. Correct method for modulus
Unit Vector = $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$	A1√ [3]	co for his vector **AB**.
(ii) $\overrightarrow{OB}.\overrightarrow{OC} - 4 + 6 - 2p$ $= 0 \rightarrow p = 5$	M1A1 [2]	Dot product = 0. co

Q7.

6 **(i)** $\mathbf{OA.OC} = -4p^2 - q^2 + 4p^2 + q^2$ $= 0$	M1 A1 [2]	Attempt scalar product. Allow M1 even for e.g. $\mathbf{OA.OB} = 2pq - 2pq$ etc.
(ii) $\mathbf{CA} = \mathbf{OA} - \mathbf{OC} = (\pm)(1 + 4p^2 + q^2)$ **(i)** $\lvert\mathbf{CA}\rvert - 1 + 4p^2 + q^2$	M1 A1 [2]	Ignore $\mathbf{CA} = \mathbf{OC} - \mathbf{OA}$ Not $\sqrt{(1+4p^2+q^2)^2}$
(iii) $\mathbf{BA} = \mathbf{OA} - \mathbf{OB} = \mathbf{i} + 6\mathbf{j} + 2\mathbf{k} - (2\mathbf{j} - 6\mathbf{k})$ $= (\pm)(\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$	M1	Allow subtn reversed for both M marks
$\dfrac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}} \to \dfrac{1}{9}(\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$	M1A1 [3]	M1 independent of 1st M1

Q8.

8	$\overrightarrow{OA} - \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix}$ and $\overrightarrow{OB} - \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$.			
	(i) $\mathbf{OC} = \mathbf{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$	M1		Knowing how to find **OC**
	Uses **OC** and **OB**	B1		Using **OC.OB** or **CO.BO**
	$\mathbf{OC.OB} = 22 = 7 \times \sqrt{29} \cos BOC$	M1 M1		M1 Use of $x_1x_2 + \ldots$ M1 for modulus
	$\to$ Angle $BOC =$ 54.3° (or 0.948 rad)	M1 A1	[6]	M1 everything linked. (nb uses **BO.OC** loses B1 A1) (nb uses other vectors – max M1M1)

(ii)	Modulus of **OC** = 7 Vector = 35 ÷ 7 × **OC**	M1		Knows to scale by factor of 35 ÷ Mod
	$\to \pm 5\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$	A1√	[2]	For their **OC.**

Q9.

9 $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}$.			
(i) Scalar product $= -18 - 48$ $-66 = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \cos\theta$ $\lvert \mathbf{a} \rvert = 7$ and $\lvert \mathbf{b} \rvert = 10$ $\rightarrow$ Angle $AOB = 160.5°$	M1 M1 M1 A1	[4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Linking everything correctly Correct modulus of either *a* or *b*. co
(ii) $\overrightarrow{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$	B1		co. allow ±.
Modulus = 6 Vector $= 5 \times \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}$	M1 A1	[3]	For modulus and multiplying by "5" co
(iii) $\begin{pmatrix} 2 \\ 3-6p \\ -6+8p \end{pmatrix} . \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix} = 0$	B1		For $\overrightarrow{OA} + p\overrightarrow{OB}$ as single vector.
$\rightarrow p = \frac{1}{2}$	M1 A1	[3]	Scalar product = 0. Co (beware fortuitous answers)

Q10.

5	$\overrightarrow{AC} = -6\mathbf{i} + 10\mathbf{k}$	B1		co (or $\overrightarrow{CA}$)
	$\overrightarrow{BC} = -8\mathbf{j} + 10\mathbf{k}$	B1		co (or $\overrightarrow{CB}$)
	$\overrightarrow{AC}.\overrightarrow{BC} = 100$	M1		Must be scalar – available for any pair
	$\overrightarrow{AC}.\overrightarrow{BC} = \sqrt{136}\sqrt{164}\cos ACB$	M1 M1		For modulus – available for any vector All linked correctly – for *ACB* only
	Angle $ACB = 48.0°$	A1	[6]	co

Q11.

10	(i)	$\overrightarrow{OA}.\overrightarrow{OB} = -6+2+12 = 8$	M1		Use of $x_1x_2 + y_1y_2 + z_1z_2$
		$\cos AOB - \frac{8}{\sqrt{14}\sqrt{29}}$	M1 M1		Mod worked correctly for either one Division of "8" by product of mods
		$AOB = 66.6°$	A1	[4]	
	(ii)	$3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + p(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	B1	[1]	In any unsimplified form
	(iii)	$\overrightarrow{BC} = \mathbf{i}(3+2p) + \mathbf{j}(-2+p) + \mathbf{k}(4-3p)$	M1		
		Their $\overrightarrow{BC}.[2\mathbf{i} + \mathbf{j} - 3\mathbf{k}] = 0$	M1		Scalar product = 0 used
		$2(3+2p) + (p-2) - 3(4-3p) = 0$	A1√		ft from their BC
		$p = 4/7$ 0.571	A1	[4]	cao

Q12.

8	(i)	$(4i + 7j - pk).(8i - j - pk) = 25 + p^2$	M1A1	[2]	$x_1x_2 + y_1y_2 + z_1z_2$ (Not $25 + (-p)^2$)
	(ii)	$25 + p^2 = 0 \Rightarrow$ no real solutions	B1√	[1]	Ft provided equation has no real solutions
	(iii)	$\cos 60 - \frac{OA.OB}{\lvert OA\rvert\lvert OB\rvert}$ used	M1		$OA.OB$ must be scalar
		$\lvert OA\rvert = \sqrt{65+p^2}$ or $\lvert OB\rvert = \sqrt{65+p^2}$	M1		Not $\sqrt{65-p^2}$ unless follows $\sqrt{65+(-p)^2}$
		$\frac{25+p^2}{65+p^2} = \frac{1}{2}$ or $\frac{his\ scalar(i)}{65+p^2} = \frac{1}{2}$	A1√		Scalar product $= 25 + p^2$ can score here if not scored in part **(i)**
		$p = \pm 3.87$ or $\pm\sqrt{15}$	A1	[4]	

Q13.

6	(i)	Scalar product = 15−8+3 10 = |**OA**| |**OB**| cos θ |**OA**| = √26, |**OB**| = √38 Angle BOA = 71.4 or 71.5 or 1.25 radians	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct magnitude for either Linking everything correctly cao
	(ii)	**a**+½(**b**–**a**) or **b**+½(**a**–**b**) or ½(**a**+**b**) –2**b** + *their* **c** oe –6**i** + 5**j** + 4**k**	M1 M1 A2,1,0 [4]	

Q14.

9 (i)	$\overrightarrow{CD} = -3\begin{pmatrix}2\\1\\2\end{pmatrix} + 2\begin{pmatrix}4\\0\\6\end{pmatrix} = \begin{pmatrix}2\\-3\\6\end{pmatrix}$	B1	
	Unit vector = $\frac{1}{7}\begin{pmatrix}2\\-3\\6\end{pmatrix}$	M1A1 [3]	Allow M1A1 for $\frac{1}{7}\begin{pmatrix}-2\\3\\-6\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{7} & \frac{-3}{7} & \frac{6}{7}\end{pmatrix}$ etc
(ii)	$\overrightarrow{OE} = \begin{pmatrix}6\\3\\6\end{pmatrix} + \begin{pmatrix}1\\-1½\\3\end{pmatrix} = \begin{pmatrix}7\\1½\\9\end{pmatrix}$	M1A1	or equivalent method
	$\overrightarrow{OE}.\overrightarrow{OD} = 56 + 0 + 108 = 164$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$\lvert\overrightarrow{OE}\rvert = \sqrt{132.25}$ (= 11.5); $\lvert\overrightarrow{OD}\rvert = \sqrt{208}$	M1	Correct method for moduli
	$164 = \sqrt{132.25} \times \sqrt{208} \times \cos\theta$	M1	All connected correctly. Dependent on $\overrightarrow{OE}, \overrightarrow{EO}, \overrightarrow{OD}, \overrightarrow{DO}$ used
	$\theta = 8.6°$ cao	A1 [6]	

Q15.

9 (i)	$p = 2$	B1	
	Unit vector $\frac{1}{\sqrt{6}}\begin{pmatrix}2\\1\\1\end{pmatrix}$ or $\frac{1}{\sqrt{24}}\begin{pmatrix}4\\2\\2\end{pmatrix}$ oe	M1 A1 √ [3]	ft for *their p*
(ii)		M1 A1	
	$5p - p^2 = 0 \Rightarrow p = 0$ or 5	M1 A1 √	ft from *their AB*
		A1 [5]	cao
(iii)	$\overrightarrow{OC} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}4\\2\\3\end{pmatrix} - \begin{pmatrix}3\\1\\1\end{pmatrix}$ oe $= \begin{pmatrix}1\\1\\2\end{pmatrix}$	M1 A1 [2]	

Q16.

3 (i)	**DB** = 6**i** + 4**j** – 3**k** cao	B1	
	DE = 3**i** +2**j** – 3**k** cao	B1 [2]	
(ii)	**DB.DE** = 18 + 8 + 9 = 35	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	|**DB**| = √61 or |**DE**| = √22	M1	Correct method for moduli
	$35 - \sqrt{61} \times \sqrt{22} \times \cos\theta$ oe	M1	All connected correctly
	$\theta - 17.2°$ (0.300 rad) cao	A1 [4]	Use of e.g. **BD. DE** can score M marks (leads to obtuse angle)

Q17.

4	**(i)**	**OD** = 4**i** + 3**j** **CD** = 4**i** + 3**j** −10**k**	B1 B1√ [2]	 √ for **OD** − **10k**
	(ii)	**OD.CD** = 9 + 16 = 25 |**OD**| = √25 or |**CD**| = √125 $25 = \sqrt{25} \times \sqrt{125} \times \cos\theta$ oe ODC = 63.4° (or 1.11 rads)	M1 M1 M1 A1 [4]	Use of $x_1 x_2 + y_1 y_2 + z_1 z_2$ Correct method for moduli All connected correctly cao